647510 **MSC1MatC102x** **Seat No :________**

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination
Nov/Dec-2023
Real Analysis(Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. Figures to the right indicate marks.
2. All questions are compulsory.

Que.1(A) Answer any two out of three. (10)

1) Define Lebesgue outer measure of set and show that Lebesgue outer measure of a finite interval is its length.
2) Prove that every closed and open set are measurable.
3) Show that countable union of measurable sets is again measurable.

Que.1(B) Answer any one out of two. (04)

1) Prove that if f and g are measurable functions then $f \cdot g$ is also measurable function.
2) If $\{A_n\}$ is a countable collection of collection of subsets of $\mathbb{R}$, then $m^*(\cup_n A_n) \leq \sum_n m^* A_n$.

Que.2(A) Answer any two out of three. (10)

1) If $f: [a, b] \to \mathbb{R}$ is bounded function then
$$\overline{\int_a^b} f(x)dx = inf_{\psi \geq \phi} \int_a^b \psi(x)dx$$
Where ψ & ϕ are step functions over $[a, b]$
2) Define Lebesgue integral of a bounded measurable function. If f and g are bounded measurable functions defined on measurable set E then show that
$$\int_E af + bg = a \int_E f + b \int_E g$$
3) State and prove monotone convergence theorem.

Que.2(B) Answer any one out of two. (04)

1) Define simple function and show that every simple function is measurable.
2) Evaluate: $\lim\limits_{n\to\infty} \int_1^a \frac{nx}{1+n^2x^2} dx$

Que.3(A) Answer any two out of three. (10)

1) If f is function of bounded variation on $[a, b]$ then $T_a^b = P_a^b + N_a^b$ and $f(b) - f(a) = P_a^b - N_a^b$
2) A function f is of bounded variation iff f is difference of two monotonically real valued increasing functions on $[a, b]$.
3) If function f is integrable on $[a, b]$ and $\int_a^b f(t)dt = 0$ for $x \in [a, b]$ then show that $f = 0$ almost everywhere in $[a, b]$.

Que.3(B) Answer any one out of two. (04)

1) If function f is bounded away from zero then $\frac{1}{f}$ is of bounded variation.
2) Let $f: [0,1] \to \mathbb{R}$ be defined as
$$f(x) = \begin{cases} \sin\left(\frac{1}{x}\right); x \neq 0 \\ 0 \qquad ; x = 0 \end{cases}$$
Then show that f is not of bounded variation on $[0,1]$

Que.4(A) Answer any two out of three. (10)

1) Show that $\|.\|_1$ is normed space on $S_1 \,/\, R$ & pair $(S_1/R\,, \|.\|_1\,)$ is normed linear space over $\mathbb{R}$ which is denoted by $L^1[0,1]$.
2) Let f be integrable function on $[a,b]$. And let $F(x) = F(a) + \int_a^b f$ for all $x \in [a,b]$ then $F'(x) = f(x)$ for almost all $x \in [a,b]$.
3) State and prove Holder's Inequality.

Que.4(B) Answer any one out of two. (04)

1) Let X be a non-empty subset of $\mathbb{R}$ and define $d: X \times X \to \mathbb{R}$ by $d(x,y) = \|x-y\|, \forall x,y \in X$ then show that d is a metric on $\mathbb{R}$ and (X,d) is metric space.
2) Define norm of a vector space and Pseudo norm (P-norm)

Que.5(A) Answer any two out of three. (10)

1) If function f is an absolutely continuous on $[a,b]$ and $f' = 0$ a.e. then f is constant function.
2) State and prove Fatou's lemma.
3) Let $f,g \in S$ and define a relation on R on S as fRg iff $f = g$ a.e. on $[0,1]$ then prove that R is an equivilence relation.

Que.5(B) Answer any one out of two. (04)

1) Show that if E is measurable set then its complement is also measurable.
2) Show that if function f is Lipschitz's function on $[a,b]$ then it is absolutely continuous.

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